Large Extra Dimensions Effects at Z pole and in Higgs Boson Production at Linear Colliders

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Summary

- Q.G. & L.E.D. and the Hierarchy Problem
- Real Kaluza Klein Gravitons Productions
- Virtual KK Graviton exchanges
  - At the Z pole
  - In Higgs boson production via WW Fusion
- Conclusions

The Hierarchy Problem

- Two fundamental scales in nature
  - Plank Mass $\mathrm{M}_{\mathrm{pl}} = 10^{19}$ GeV
  - Electroweak scale $\Lambda_{\mathrm{EW}} = < \mathrm{H} > = 240$ GeV
- 17 order of magnitudes of difference !
- Difficult to mantain at quantum level
- Quadratic divergencies in scalar sector

  ($\delta m_H^2 = \alpha \Lambda_{UV}^2 + \dots$ fine tuning problem)
- Supersymmetry solves this problem

  boson $\leftrightarrow$ fermion symmetry
- Standard approach: Fundamental theory is at M_{pl} and Λ_{EW} is derived as a small number from dynamics

Q-Gravity in Large Extradimensions

Arkani-Hamed, Dimopoulos, Dvali ('98)
A new approach to the hierarchy problem

Λ_{EW} is a fundamental scale
M_{pl} is a derived effect from geometry

This requires

- **Compact Extra Spatial Dimensions**
- **Confinement of matter on subspaces**

The idea: **Matter fields confined in the usual 4 dimensional space, Gravity propagating in $D = 4 + \delta$ space**

Weakness of gravity: **Due to large compact spatial dimensions**

$$G_N = \frac{G_N^D}{V_{D-4}}$$

Newton law modified at small distances

- $r > R$ $$\mathbf{F_N(r) = G_N \frac{m_1 m_2}{r^2}}$$

- $r < R$ $$\mathbf{F_N(r) = G_N R^{\delta} \left(\frac{m_1 m_2}{r^{2+\delta}} \right)}$$

The potential

$$\mathbf{V(r) = G_N \frac{m_1 m_2}{r} \left(1 + C e^{-\frac{r}{\lambda}} \right)}$$

Plank masses

$$G_N = \frac{1}{M_{pl}^2} \qquad G_N^D = \frac{1}{M_D^{D-2}}$$

$$M_{pl} = \frac{1}{\sqrt{8\pi}} M_D \left(R M_D\right)^{\frac{D-4}{2}}$$

If we want $M_D \simeq 1$ TeV, then R must be large in units of M_D^{-1}.

$D = 4+\delta$, $M_D \simeq 1$ TeV $\Rightarrow$ $R \simeq 10^{\frac{30}{\delta}-17}$ cm

- $\delta = 1 \qquad R = \left(5 \times 10^{-4}\ \text{eV}\right)^{-1} \simeq 10^8$ **km**
- $\delta = 2 \qquad R = \left(5 \times 10^{-4}\ \text{eV}\right)^{-1} \simeq 0.4$ **mm**

Since Newton Law has NOT been tested down the mm scale $\Rightarrow$ number of extradimensions $\delta \geq 2$

Kaluza-Klein exitations in 4D

- Massive KK Gravitons

$$L_{int}(x) \sim \frac{1}{M_{pl}} \Sigma n \, G^{(n)}_{\mu\nu}(x) T^{\mu\nu}(x)$$

- Massive KK Graviscalars

$$L_{int}(x) \sim \frac{1}{M_{pl}} \Sigma n \, S^{(n)}(x) T^{\mu}_{\mu}(x)$$

$T^{\mu}_{\mu}(x)$ Energy Momentum Tensor

Masses of KK excitations $M_n^2 \sim \frac{n^2}{R^2}$

Inclusive production of KK Gravitons

- Mass splitting

$$\mathbf{\Delta m \simeq \frac{1}{R} = M_D \left(\frac{M_D}{M_{pl}}\right)^{2/\delta}}$$

$\Delta m = 20\text{KeV}$ $(\delta = 4)$

$\Delta m = 7\text{MeV}$ $(\delta = 6)$

- For small $\delta \leq 6$:

  Large number of accessible KK modes.

  $\sum_{\mathbf{n}} \longrightarrow \int \mathbf{dN}$

$$dN = A_{\delta-1} \left(\frac{\bar{M}_{Pl}^2}{M_D^{2+\delta}}\right) m_G^{\delta-1} dm_G$$

  $A_{\delta-1}$ area of δ-sphere, m_G graviton mass

- Inclusive production of KK Gravitons

$$\mathbf{(E/M_{pl})^2} \quad \Rightarrow \quad \mathbf{(E/M_D)^{2+\delta}}$$

Bounds from Astrophys./Cosmology

SuperNova SN1987A collapse

- Nucleon-Nucleon Brehmstrahlung (T$\simeq$ 30 MeV)

$$\mathcal{L}_{grav} \simeq M_{SN} \times \frac{n_N^3}{\rho} \times (30\text{millibarn}) \times \left(\frac{T}{M_D}\right)^{\delta+2}$$

$$\mathcal{L}_{grav} \leq 10^{53}\text{erg}s^{-1} \simeq (10^{16}\text{GeV})^2 \quad \Rightarrow \quad M_D > 10^{\frac{15-4.5\delta}{\delta+2}}\ \text{TeV}$$

- $\mathbf{M_D > 30}$ **TeV** $\mathbf{(\delta = 2)}$ $\mathbf{M_D > 2}$ **TeV** $\mathbf{(\delta = 3)}$

Cosmic γ-Rays

$\mathbf{G \to \gamma\gamma}$ $\quad \tau \simeq \mathbf{3 \times 10^9}\text{yrs}\left(\frac{\mathbf{100}\text{MeV}}{\mathbf{m_G}}\right)^{\mathbf{3}}$

Gravitons sources

- Thermal production after nucleos :

  $\mathbf{M_D > 110}$ **TeV** $\mathbf{(\delta = 2)}$ $\mathbf{M_D > 5}$ **TeV** $\mathbf{(\delta = 3)}$

- Neutron stars :

  $\mathbf{M_D > 450}$ **TeV** $\mathbf{(\delta = 2)}$ $\mathbf{M_D > 30}$ **TeV** $\mathbf{(\delta = 3)}$

Bounds from accelerator experiments

- **KK gravitons production at high energy**
  G. Giudice, R. Rattazzi, and J.D. Wells ('98)
  - LEP and future e^+e^- linear colliders
    $\mathrm{e^+e^-} \to \gamma + \mathrm{G}$ 	G detected as missing energy
  - Hadron colliders (elementary processes)
    $\mathrm{q\bar{q}} \to \mathrm{g} + \mathrm{G}$ 	$\mathrm{q\ g} \to \mathrm{g} + \mathrm{G}$ 	$\mathrm{g\ g} \to \mathrm{g} + \mathrm{G}$

- **Precision Observables:**
  - $(g-2)$ of muon 	M.L. Graesser (2000)
  - precision EW observables at LEP 	ADD ('98)
  - four-fermion interactions
    ADD ('98), G. Giudice, R. Rattazzi, J.D. Wells ('98)

- **Rare decays to Kaluza-Klein Gravitons**
  - $\mu \to \nu_\mu \mathrm{e} \bar{\nu}_\mathrm{e} + \mathrm{G}$ 	$\mathrm{K} \to \pi + \mathrm{G}$
    $\mathrm{J/Psi} \to \gamma + \mathrm{G}$ 	$\Upsilon \to \gamma + \mathrm{G}$ 	**J.Bijnens and M.Maul (2000)**
  - $\mathrm{Z} \to \mathrm{f\bar{f}} + \mathrm{G}$ 	$\mathrm{t} \to \mathrm{bW} + \mathrm{G}$ 	$\mathrm{H} \to \mathrm{f\bar{f}} + \mathrm{G}$
    $\mathrm{H} \to \mathrm{W^+W^-} + \mathrm{G}$ 	**E.G. and B.Mele (2002)**

Experimental Bounds on M_D

$(\sigma_{\text{Signal}} > 5\sqrt{\sigma_B \mathcal{L}}/\mathcal{L})$

G. Giudice, R. Rattazzi, and J.D. Wells ('98)

$e^+e^- \to \gamma + G$ G as missing energy

- **LEP200**, $\mathcal{L} = 4 \times 500\ \text{fb}^{-1}$

  $\delta = 2 \quad M_D > 1.3$ TeV

- e^+e^- **LC** ($\sqrt{s}$=1 TeV), $\mathcal{L}$=200 fb^{-1}

  $\delta = 2 \quad M_D > 4.1$ TeV

$pp \to \text{Jets} + G$ G as missing energy

- **LHC** , $\mathcal{L} = 100\ \text{fb}^{-1}$

  $\delta = 2 \quad M_D > 8.5$ TeV

Bounds from precision observables

g-2 of the muon

Graesser (2000)

- It is very sensitive to new physics

- Contribution of each KK graviton to g-2 is finite. $\sum$ over KK states is divergent!

$$\Delta a_\mu = \frac{7}{12\pi}\left(\frac{m_\mu}{M_D}\right)^2 \times \lambda \qquad (\delta = 2)$$

$$\Delta a_\mu = \frac{25\pi}{288}\left(\frac{m_\mu}{M_D}\right)^2 \times \lambda \qquad (\delta = 6)$$

λ parametrizes unknown UV physics contributions at M_D

- Δa_μ scales as $1/M_D^2$ independently on δ.

$$\mathbf{\Delta a_\mu^{\text{Ext.Dim}} = (2, 3) \times 10^{-9} \left(\frac{TeV}{M_D}\right)^2 \times \lambda}$$

$(\delta = 2)$ $(\delta = 6)$

BNL E821 $\Rightarrow \Delta a_\mu < 4 \times 10^{-10}$ at 2σ level

$$\mathbf{M_D < 2.2\ (2.7)\ \text{TeV}\ \sqrt{\lambda}}$$

Rare decays to KK gravitons

- Decay $\mathbf{K \to \pi + G}$ ADD ('98)

  Due to angular momentum conservation ⇒ spin-0 KK gravitons

$$\Gamma_{K\to\pi+G} \simeq \frac{1}{16\pi}\left(\frac{m_K^5}{m_W^4}\right) \times \left(\frac{m_K}{M_D}\right)^{\delta+2}$$

  $\mathrm{BR} < \mathbf{10^{-12}}$ (for $\delta = 2$ and $M_D = 1$ TeV)

- $\mathbf{Quarkonium(q\bar{q}) \to \gamma + G}$

  Bijnens and Maul (2000)

  Y decays are more favorable than $J/\Psi \to \gamma + G$

  $\mathbf{Y \to \gamma + G}$ spin-2 dominated

  $\mathrm{BR}^{\mathrm{exp}}(Y \to \gamma + G) < 10^{-5}$

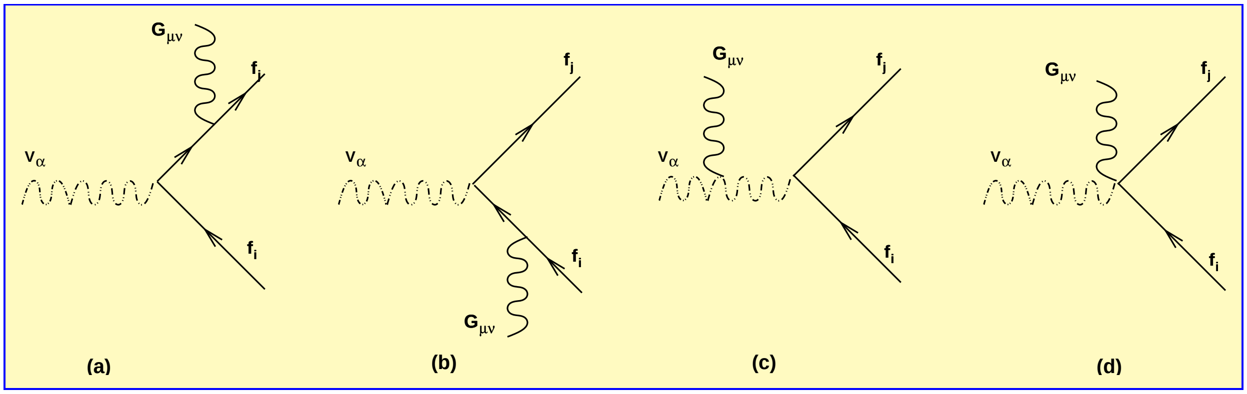
  $\mathbf{M_D > 50 \quad (9)\ GeV}$ for $\delta = 2 \quad (6)$

- $\mathbf{Z \to f\bar{f} + G}$ ⇒ strong bounds !

  Balázs, et al. ('99), E.G. and Mele (2002)

Vector decay $\mathrm{V \to f\bar{f} + G}$

E.G. and B.Mele, Nucl.Phys. B647, 319 (2002)



Sum over polarizations of graviton

$$\sum_{\sigma=1}^{5} \epsilon_{\mu\nu}(k,\sigma)\, \epsilon_{\alpha\beta}(k,\sigma) = P^{G}_{\mu\nu\alpha\beta}$$

$$\begin{aligned} P^{G}_{\mu\nu\alpha\beta} &= \frac{1}{2}\left(\eta_{\mu\alpha}\eta_{\nu\beta} + \eta_{\mu\beta}\eta_{\nu\alpha} - \eta_{\mu\nu}\eta_{\alpha\beta}\right) \\ &- \frac{1}{2\,m_G^2}\left(\eta_{\mu\alpha}k_\mu k_\beta + \eta_{\nu\beta}k_\mu k_\alpha + \eta_{\mu\beta}k_\nu k_\beta + \eta_{\nu\alpha}k_\mu k_\beta\right) \\ &+ \frac{1}{6}\left(\eta_{\mu\nu} + \frac{2}{m_G^2}k_\mu k_\nu\right)\left(\eta_{\alpha\beta} + \frac{2}{m_G^2}k_\alpha k_\beta\right), \end{aligned}$$

- Due to energy-momentum conservation, no graviton mass singularity in the final amplitude. $\lim_{(\mathbf{m_G \to 0})} \sum_{\text{pol}} |\mathrm{M}|^2 =$ **finite**

- However, for $m_G \to 0$, we have

  van-Dam Veltman discontinuity

Inclusive total width $V \to f\bar{f} + G$

- $\Gamma(V \to f\bar{f} + G_X) = N_f \frac{M_V^3 \, G_V \, S_{\delta-1}}{96\,\pi^3\,\sqrt{2}} \left(\frac{M_V}{M_D}\right)^{2+\delta} \times$

$$\left\{ |g_{VA}^{(+)}| \; I_V^{(+)}(x_\Delta, x_f, \delta) + |g_{VA}^{(-)}| \; I_V^{(-)}(x_\Delta, x_f, \delta) \right\}$$

$G_V = g^2/(4\sqrt{2}M_V^2)$, $g_{VA}^{\pm} = g_V^2 \pm g_A^2$, $x_\Delta = \Delta_{exp}^2/m_V^2$, $x_a = m_a^2/m_V^2$, where Δ_{exp} is the experimental resolution on missing energy.

Unitarity condition

$$\frac{\Gamma(V \to f\bar{f} + G_X)}{\Gamma(V \to f\bar{f})} < 1$$

$$M_V < M_D \left(\frac{64\,\pi^2}{I_V^{(+)}(x_\Delta, 0, \delta)\, S_{\delta-1}} \right)^{\frac{1}{2+\delta}}$$

$\delta = (2, 3, 4, 5, 6)$

$$M_V < M_D \times (5.4,\; 4.7,\; 4.2,\; 3.8,\; 3.5)$$

Bounds on M_D from Z, W, t, H decays

E.G. and B.Mele, Nucl.Phys. B647, 319 (2002)

$$\mathrm{BR}(\mathrm{Z} \to \sum_{\mathrm{f}} \mathrm{f\bar{f}} + \mathrm{G_X}) = 8.2 \times 10^{-8} \left(\frac{\mathrm{TeV}}{\mathrm{M_D}}\right)^4 \qquad (\delta = 2)$$

$$\mathrm{BR}(\mathrm{W} \to \sum_{\mathrm{f',f}} \mathrm{f'\bar{f}} + \mathrm{G_X}) = 5 \times 10^{-8} \left(\frac{\mathrm{TeV}}{\mathrm{M_D}}\right)^4 \qquad (\delta = 2)$$

$$\mathrm{BR}(\mathrm{t} \to \mathrm{W\,b} + \mathrm{G_X}) = 1.8 \times 10^{-7} \left(\frac{\mathrm{TeV}}{\mathrm{M_D}}\right)^4 \qquad (\delta = 2)$$

Higgs decays $(\delta = 2)$ $\qquad$ $\mathrm{m_H}$

$$\mathrm{BR}(\mathrm{H} \to \mathrm{\bar{b}b} + \mathrm{G_X}) = 2.2 \times 10^{-7} \left(\frac{\mathrm{TeV}}{\mathrm{M_D}}\right)^4 \qquad 120 \text{ GeV}$$

$$\mathrm{BR}(\mathrm{H} \to \mathrm{WW} + \mathrm{G_X}) = 2.1 \times 10^{-5} \left(\frac{\mathrm{TeV}}{\mathrm{M_D}}\right)^4 \qquad 500 \text{ GeV}$$

Virtual KK Gravitons exchange

- The KK Gravitons exchange induces local dim. 8 operators $(T_{\mu\nu}T^{\mu\nu})$:
  $e^-e^+ \to f\bar{f}$, $e^-e^+ \to \gamma\gamma$, $W^+W^- \to W^+W^-$, etc.

- Real part of amplitude $\mathbf{Re[A]}$ is divergent.
  Predictivity in terms of M_D is lost.

- Imaginary part $\mathbf{Im[A]}$ is finite (due to branch-cut singularity of real gravitons emission)

- Resonant SM processes can have interference with $\mathbf{Im[A]}$ of the graviton exchange
  Finite result predicted in terms of M_D.

- Virtual Graviton exchanges at the Z-pole in large Extra Dimensions
  A. Datta, E.G, and B. Mele, PLB 552 (2003) 237

Scattering amplitude (s-channel)

$$\mathcal{A} = \frac{1}{\bar{M}_P^2} \sum_n \left\{ T_{\mu\nu} \frac{P^{\mu\nu\alpha\beta}}{s - m_{G_n}^2} T_{\alpha\beta} + \frac{1}{3}\left(\frac{\delta - 1}{\delta + 2}\right) \frac{T_\mu^\mu T_\nu^\nu}{s - m_{S_n}^2} \right\}$$

Graviton projector (unitary gauge)

$$P^{\mu\nu\alpha\beta} = \frac{1}{2}\left(\eta^{\mu\alpha}\eta^{\nu\beta} + \eta^{\mu\beta}\eta^{\nu\alpha}\right) - \frac{1}{3}\eta^{\mu\nu}\eta^{\alpha\beta} + \ldots$$

$T_{\mu\nu}$ does not depend on n $\rightarrow$ Factorization

$$\mathcal{A} = \mathcal{S}(\mathbf{s}) \left(\mathbf{T}_{\mu\nu}\mathbf{T}^{\mu\nu} - \frac{1}{\delta + 2}\left(\mathbf{T}_\mu^\mu\right)^{\mathbf{2}} \right)$$

$$\mathcal{S}(\mathbf{s}) = \frac{\mathbf{1}}{\bar{\mathbf{M}}_\mathbf{P}^\mathbf{2}} \sum\nolimits_\mathbf{n} \frac{\mathbf{1}}{\mathbf{s} - \mathbf{m}_\mathbf{n}^\mathbf{2}}$$

$\sum_n \rightarrow \int$

$$\mathcal{S}(\mathbf{s}) = \frac{1}{M_D^{2+\delta}} \int \mathbf{d}^\delta \mathbf{q_T} \frac{1}{\mathbf{s} - \mathbf{q_T^2}} = \frac{\pi^{\frac{\delta}{2}}}{M_D^4} \mathbf{\Gamma}(\mathbf{1} - \frac{\delta}{2}) \left(-\frac{s}{M_D^2}\right)^{\frac{\delta}{2} - 1}$$

- Real part is divergent
- **Imaginary part is finite**

$$\mathbf{Im}[\mathcal{S}(\mathbf{s})] = \frac{A_{\delta-1}}{2} \frac{\mathbf{s}^{\frac{\delta - 2}{2}}}{\mathbf{M_D^{2+\delta}}}$$

Gravitational interference at Z pole

A. Datta, E.G, and B. Mele, PLB 552 (2003) 237

$$\frac{d\sigma_f}{d\cos\theta} = \frac{9\pi}{2M_Z^2}\frac{\Gamma_{e^+e^-}\Gamma_{f\bar{f}}}{\Gamma_Z^2} S_f(\cos\theta)$$

$$\mathbf{S_f(x) = 1 + x^2 - \Delta_1\left(1 - 3x^2\right) + 2A_eA_f\left(x + \Delta_2 x^3\right)}$$

$$\Delta_1 = R_\delta\,\frac{A_e A_f}{g_V^e g_V^f}, \qquad \Delta_2 = \frac{R_\delta}{g_A^e g_A^f},$$

$$R_\delta = \frac{\pi S_{\delta-1}}{32\sqrt{2}G_F M_Z^2}\left(\frac{\Gamma_Z}{M_Z}\right)\left(\frac{M_Z}{M_D}\right)^{2+\delta}$$

- **New contribution: vanishes** when integrated over the full range of $\cos\theta = (-1, 1)$

- $\delta = 2$, $M_D = 1$ **TeV** $\Rightarrow$ $\Delta_{1,2} \simeq \mathcal{O}(10^{-4})$ **too small for present EXP. sensitivity**

Lepton final states ($\delta = 2$)

M_D	Δ_1	Δ_2	Δ_1'
0.5	2.2×10^{-3}	5.4×10^{-4}	1.1×10^{-3}
1	1.3×10^{-4}	3.4×10^{-5}	6.8×10^{-5}
1.5	2.7×10^{-5}	6.7×10^{-6}	1.3×10^{-5}
2	8.4×10^{-6}	2.1×10^{-6}	4.2×10^{-6}
2.5	3.4×10^{-6}	8.7×10^{-7}	1.7×10^{-6}
3	1.7×10^{-6}	4.2×10^{-7}	8.4×10^{-7}

Up-Quarks final states ($\delta = 2$)

M_D	Δ_1	Δ_2	Δ_1'	Δ_2'
0.5	-1.9×10^{-3}	-5.4×10^{-4}	-9.5×10^{-4}	-1.1×10^{-3}
1	-1.2×10^{-4}	-3.4×10^{-5}	-5.9×10^{-5}	-6.8×10^{-5}
1.5	-2.3×10^{-5}	-6.7×10^{-6}	-1.2×10^{-5}	-1.3×10^{-5}
2	-7.4×10^{-6}	-2.1×10^{-6}	-3.7×10^{-6}	-4.2×10^{-6}
2.5	-3.0×10^{-6}	-8.7×10^{-7}	-1.5×10^{-6}	-1.7×10^{-6}
3	-1.5×10^{-6}	-4.2×10^{-7}	-7.3×10^{-7}	-8.4×10^{-7}

Down-Quarks final states ($\delta = 2$)

M_D	Δ_1	Δ_2	Δ_1'	Δ_2'
0.5	1.5×10^{-3}	5.4×10^{-4}	7.4×10^{-4}	1.1×10^{-3}
1	9.1×10^{-5}	3.4×10^{-5}	4.6×10^{-5}	6.8×10^{-5}
1.5	1.8×10^{-5}	6.7×10^{-6}	9.1×10^{-6}	1.3×10^{-5}
2	5.7×10^{-6}	2.1×10^{-6}	2.9×10^{-6}	4.2×10^{-6}
2.5	2.3×10^{-6}	8.7×10^{-7}	1.2×10^{-6}	1.7×10^{-6}
3	1.1×10^{-6}	4.2×10^{-7}	5.7×10^{-7}	8.4×10^{-7}

Virtual KK Gravitons in Higgs production

A. Datta, E.G, and B.Mele, hep-ph/0303259

Light Higgs

- **Hadron Colliders**: $gg \to H \to \gamma\gamma$ **(1-loop)^2**
- Interference with $Im[A(gg \to G, S \to \gamma\gamma)]$
- Quite small effect, being coupled to the Trace$[T^\mu_\mu]$

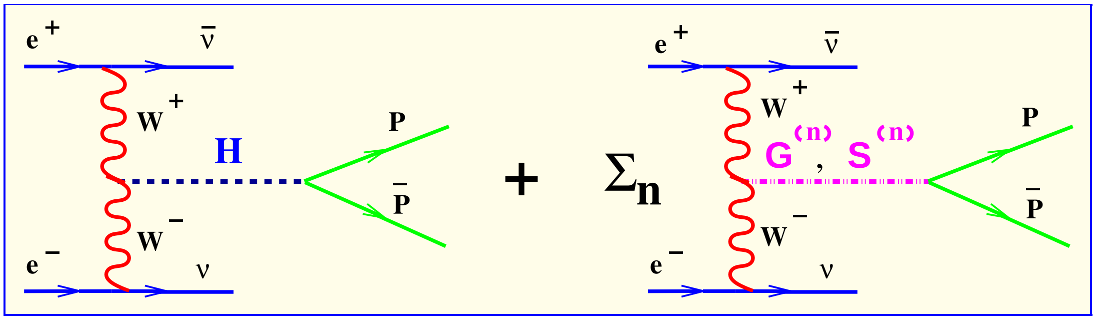
  For massless final states $T^\mu_\mu \propto \mathcal{O}(\alpha_S, \alpha_e)$ gravitational anomalies.

Heavy Higgs

- $\mathrm{e}^+\mathrm{e}^-$ **Colliders**: $WW \to H \to WW$ **Tree!**
- Large interference $Im[A(WW \to G, S \to WW)]$

  since $T^\mu_\mu \propto M_W^2$
- $\mu^+\mu^-$ **Colliders**: $\mu^+\mu^- \to H \to (WW),\ (\bar{t}t)$

Vector boson fusion in Higgs production



$P = W, Z, t$

$\mathbf{WW \to H \to P\bar{P}} \; + \; \sum_n \; \mathbf{WW \to G^{(n)}, S^{(n)} \to P\bar{P}}$

SM + Interf. with KK Gravitons exchange

$$\frac{d\sigma_\lambda^P}{d\cos\theta} = \frac{\bar{\sigma}_\lambda^P}{2} \left\{1 + \Delta_0^P + \Delta_{2,\lambda}^P \left(1 - 3\cos^2\theta\right)\right\}$$

$(x = \cos\theta)$ $\quad \Delta_0^P =$ Graviscalars $\quad \Delta_{2,\lambda}^P =$ Gravitons

$\lambda = T, L$ are the W polarizations

- **SM cross section**

$$\bar{\sigma}_\lambda^P = \frac{1}{16\pi\hat{s}} \frac{g^4 m_W^4 \xi_P}{(\hat{s} - m_H^2)^2 + m_H^2\Gamma_H^2} \sqrt{\frac{\hat{s} - 4m_P^2}{\hat{s} - 4m_W^2}}\, \rho_\lambda^P \left(\frac{\hat{s}}{m_W^2}\right)$$

At Higgs mass peak $\sqrt{\hat{s}} = m_H$

$$\mathbf{\Delta_0^P} = \mathbf{R}_\delta\, \mathbf{c_P} \left(\frac{\delta - 1}{\delta + 2}\right)$$

$$\mathbf{\Delta_{2,\lambda}^P} = \mathbf{R}_\delta\, \mathbf{f_\lambda^P} \left(\frac{\mathbf{m_H^2}}{\mathbf{m_W^2}}\right)$$

$c_W = \frac{4}{3} \qquad c_Z = \frac{2}{3} \qquad c_t = \frac{4}{3}$

$$\mathbf{R}_\delta \sim \frac{A_{\delta-1}}{\mathbf{G_F M_D^2}} \left(\frac{\mathbf{m_H}}{\mathbf{M_D}}\right)^\delta \left(\frac{\mathbf{\Gamma_H}}{\mathbf{m_H}}\right)$$

$\mathbf{W_\lambda W_\lambda \to H \to WW} \qquad (\lambda = L, T)$

$m_H(GeV)$	$\Gamma_H(GeV)$	$\Delta^W_{2,T}$	$\Delta^W_{2,L}$	Δ^W_0
300	8.5	6.7×10^{-4}	-5.1×10^{-4}	2.6×10^{-4}
500	67.5	7.7×10^{-3}	-4.4×10^{-3}	3.3×10^{-3}
800	308.7	5.2×10^{-2}	-2.7×10^{-2}	2.4×10^{-2}

$\mathbf{W_\lambda W_\lambda \to H \to t\bar{t}}$

$m_H(GeV)$	$\Gamma_H(GeV)$	$\Delta^t_{2,T}$	$\Delta^t_{2,L}$	Δ^t_0
400	28.8	-4.5×10^{-3}	2.7×10^{-3}	1.1×10^{-3}
500	67.5	-1.3×10^{-2}	7.8×10^{-3}	3.3×10^{-3}
800	308.7	-9.7×10^{-2}	5.0×10^{-2}	2.4×10^{-2}

$e^+e^-(WW) \to \nu\bar{\nu}P\bar{P}$

- The Graviton interference weighted by $(1 - 3\cos^2\theta)$ vanishes

- Graviscalar interference effects survive

- To pin down Gravitons effects $\to$ angular cuts, new asymmetries

- The $e^+e^-(WW) \to \nu\bar{\nu}P\bar{P}$ cross section

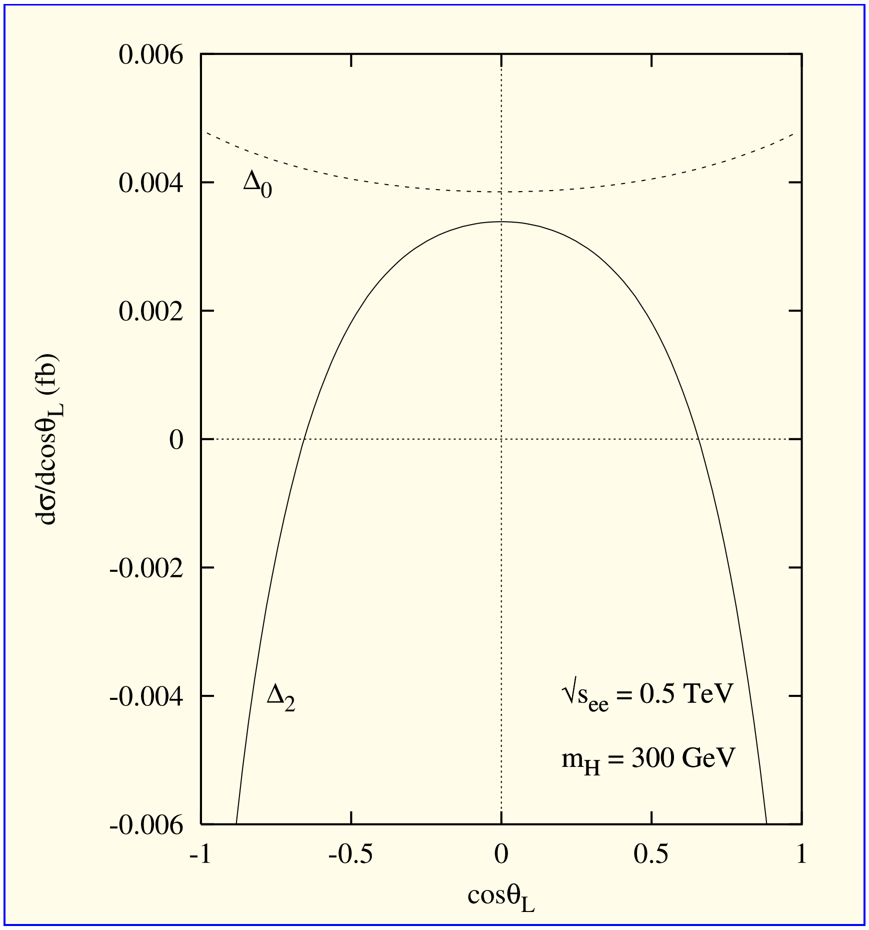
$$\frac{d\sigma_{ee}^{P}(S)}{d\cos\theta_L} = \Sigma_{\lambda=L,T} \int dx_1\, dx_2 \left\{ P_\lambda^W(x_1)\, P_\lambda^W(x_2)\, \frac{d\sigma_\lambda^P(\hat{s})}{d\cos\theta_L} \right\}$$

$\hat{s} = x_1 x_2 S$, $\quad \theta_L = P$ scattering angle in Lab. frame

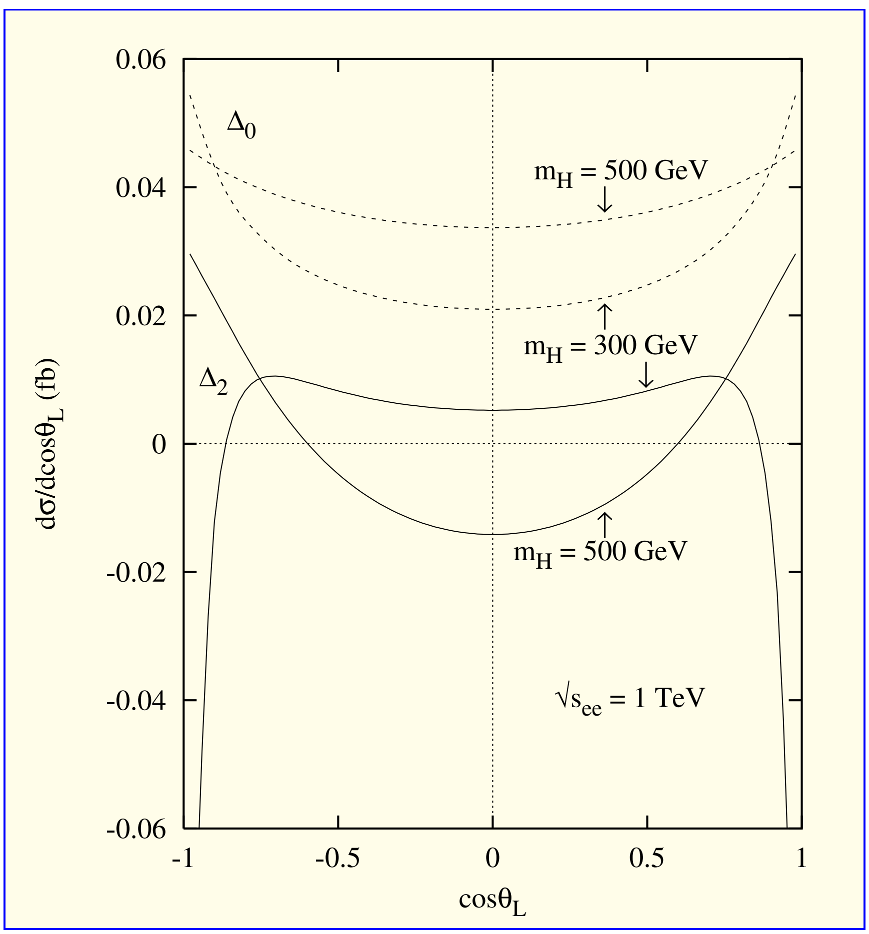
$P_\lambda^W(x) =$ W fluxes

- Approx. of Breight-Wigner with delta-Dirac

$$\frac{1}{(\hat{s} - m_H^2)^2 + m_H^2 \Gamma_H^2} \longrightarrow \frac{\pi}{m_H \Gamma_H}\, \delta(\hat{s} - m_H^2)$$



$\sqrt{S} = 500$ GeV



$\sqrt{S} = 1$ TeV

Optimized angular cuts are applied to enhance KK gravitons effect

$|\cos\theta_L| < \epsilon$

$$\sigma_{\mathbf{ee}}^{\mathbf{P},\epsilon} = \bar{\sigma}_{\mathbf{ee}}^{\mathbf{P},\epsilon} \left(1 + \boldsymbol{\Delta}_{\mathbf{0}}^{\mathbf{P}} + \alpha_{\mathbf{L}}^{\epsilon}\ \boldsymbol{\Delta}_{\mathbf{2,L}}^{\mathbf{P}} + \alpha_{\mathbf{T}}^{\epsilon}\ \boldsymbol{\Delta}_{\mathbf{2,T}}^{\mathbf{P}}\right)$$

$\bar{\sigma}_{ee}^{P,\epsilon}$ = SM contribution with angular cuts

$\sqrt{S} = 500$ GeV $m_H = 300$ GeV $\epsilon \simeq 0.66$

$\alpha_T \simeq 0.36$ $\alpha_L \simeq 0.16$

$\sqrt{S} = 1$ TeV $m_H = 500$ GeV $\epsilon \simeq 0.60$

$\alpha_T \simeq 0.13$ $\alpha_L \simeq 0.43$

- $\alpha_{L,T}^{\epsilon}$ do NOT depend on the final state

- Largest Grav. Effect is in $H \to \bar{t}t$

$$\Downarrow$$

- $m_H = 500$ GeV Grav-Interf/SM $\simeq 0.5$ %
- $m_H = 800$ GeV Grav-Interf/SM $\simeq \mathcal{O}(\%)$

Conclusions

- In ADD scenario, we analyzed Higgs production via **Vector Boson fusion** in $e^+e^-(WW) \to \nu\bar{\nu}P\bar{P}$ where $P = W, Z, t$

- The **Re[A]** of Amplitude mediated by KK Graviton exchanges is **divergent**, while **Im[A]** (s-channel) is **finite** and predictable in terms of $\mathrm{M_D}$

- Near SM resonant regions, interference of SM amplitude with **Im[A]** dominates with respect to interference with **Re[A]**

- For $\mathbf{m_H = 500 - 800}$ **GeV**, Gravit. interference effects ($\delta = 2$, $M_D = 1$TeV) are $\mathcal{O}(\%)$ on the total SM cross section.

- We also analyzed the same effect at muon collider, via $\mu^+\mu^- \to H \to P\bar{P}$. Gravitational effects are quite large. For $P = t$ and $m_H = 800$ GeV, SM deviations can reach $\simeq 10\%$ effect