Quantum Gravity in Large Extra Dimensions

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Summary

- Introduction to Extra Dimensions
  - Kaluza Klein Theories
  - Unification of Gauge and Gravitational Forces
- Extra Dimensions and the Hierarchy Problem
- Quantum Gravity in Large Extra Dimensions
- Experimental Signals at Colliders
- Conclusions

Space Dimensions and Unification

- Connection between unification of forces and number of dim.

- Electric $\vec{\mathbf{E}}$ and Magnetic $\vec{\mathbf{B}}$ fields unified in continuum 4D space-time (recognized first by Minkowski)

- Four vector potential $A_\mu = (\Phi, \vec{\mathbf{A}})$

$$\vec{\mathbf{E}} = -\vec{\nabla}\phi - \frac{\partial \vec{A}}{\partial t}, \qquad \vec{\mathbf{B}} = \vec{\nabla} \times \vec{\mathbf{A}}$$

- e.m. fields $\vec{\mathbf{E}}, \vec{\mathbf{B}}$ embedded in $F_{\mu\nu}$

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu = \begin{pmatrix} \mathbf{0} & -\mathbf{E_x} & -\mathbf{E_y} & -\mathbf{E_x} \\ \mathbf{E_x} & \mathbf{0} & \mathbf{B_z} & -\mathbf{B_y} \\ \mathbf{E_y} & -\mathbf{B_z} & \mathbf{0} & \mathbf{B_x} \\ \mathbf{E_z} & \mathbf{B_y} & -\mathbf{B_x} & \mathbf{0} \end{pmatrix}$$

$$L = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu}$$

- char. density and current $\rho, \vec{\mathbf{J}} \rightarrow J_\mu = (\rho, \vec{\mathbf{J}})$

- Maxwel's eqs $\rightarrow \partial_\mu F^{\mu\nu} = J^\nu$

Unification of E.M. and Gravity

(history of attempts)

- **1912: Gunnar Nordström** proposes gravity with scalar field coupled to trace of energy-momentum tensor T_μ^μ.

- **1914: G.N.** introduces a 5-dim A_μ to describe both EM and Gravity- (work ignored and recently rediscovered).

- **1919: Theodor Kaluza** writes a 5-dim theory for EM and Gravity. Einstein suggests publication 2 years later.

- **1926: Oscar Klein** rediscovers the theory, gives geometrical interpretation and finds **charge quantization.**

Now theory is known as **Kaluza-Klein** theory.

E.M.

Action for free fermions

$$\mathsf{S}_{\mathsf{Dirac}} = \int d^4x \; \bar{\psi} \left(i\gamma^\mu \partial_\mu - m \right) \psi$$

invariant under global gauge transformation

$\psi \rightarrow e^{i\Lambda}\psi$

Gauge Principle:

E.M. action can be obtained by requiring invariance under **local gauge transformations** $\Lambda \rightarrow \Lambda(x)$.

$$\mathsf{S}_{\mathsf{EM}} = \int d^4x \; \left[\bar{\psi} \left(i\gamma^\mu \mathsf{D}_\mu - m \right) \psi - \frac{1}{4} F_{\mu\nu} F^{\mu\nu} \right]$$

$$\mathbf{D}_\mu = \partial_\mu - iA_\mu, \qquad F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$$

local gauge transf. $= \begin{cases} \psi \rightarrow e^{i\Lambda(x)}\psi \\ A_\mu \rightarrow A_\mu + \partial_\mu \Lambda(x) \end{cases}$

A_μ is a dynamical field (**photon**)

GRAVITY

In General Relativity the dynamical variable is the space-time metric $g_{\mu\nu}$

- $g_{\mu\nu}$ is a 4×4 symmetric tensor, describing space-time geometry

$$ds^2 = g_{\mu\nu}\; dx^\mu dx^\nu$$

- **Gravitons ($h_{\mu\nu}$):** elementary quantum fluctuation of $g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}$ around Minkowski (flat) metric $\eta_{\mu\nu} = \mathrm{diag}(1, -1 -1 -1)$.

- Dynamics described by **Einstein action**

$$\mathrm{S_G} = \frac{1}{16\pi \mathrm{G_N}} \int d^4x\; \sqrt{-g}\; \mathrm{R}(g)$$

- $\mathbf{G_N}$ is the Newton's constant

- **R** is the curvature (function of the metric)

- $\mathbf{S_G}$ invariant under general coordinate transformations

Unification of EM and Gravity

Kaluza's idea is to consider General Relativity in **5-dim**

$$\hat{\mathsf{S}}_{\mathsf{G}} = \frac{1}{16\pi \hat{\mathsf{G}}_{\mathsf{N}}} \int d^5x \, \sqrt{-\hat{g}} \, \mathsf{R}(\hat{g})$$

where $\hat{g}$

$$\hat{g}_{MN}(\hat{x}) \;\; = \;\; \begin{pmatrix} g_{\mu\nu} + k^2 \Phi A_\mu A_\nu & k\Phi A_\mu \\ & \\ k\Phi A_\mu & \Phi \end{pmatrix} (\hat{x})$$

- dynamical fields $\hat{g}_{MN} \rightarrow g_{\mu\nu} \ , \ \ A_\mu \ , \ \ \Phi$

- $\hat{x}$ is a 5-dim coordinate of space. x_5 is periodic or compactified.

Kaluza-Klein modes

- Assume space is $M_4 \times S_1$

$$x_5 + 2\pi R = x_5$$

- All fields can be expanded in Fourier modes

$$\varphi(x, x_5) = \sum_{n=-\infty}^{\infty} \frac{1}{\sqrt{2\pi R}} \varphi^{(\mathbf{n})}(x)\ e^{i\frac{\mathbf{n}x_5}{R}}$$

- **Equation of motion** (scalar field)

$$\left(\eta_{MN}\partial^M\partial^N + m_0^2\right)\varphi(x, x_5) = 0$$

- **Fourier transform** $\hat{\varphi}^{(\mathbf{n})}$ of each mode $(\mathbf{n})$ satisfies (in unity $\hbar, c = 1$)

$$\left[E^2 - \vec{\mathbf{p}}^2 - \left(m_0^2 + \frac{\mathbf{n}^2}{R^2}\right)\right]\hat{\varphi}^{(\mathbf{n})} = 0$$

- mass of nth KK mode is

$$\mathbf{m_n^2} = m_0^2 + \frac{\mathbf{n}^2}{R^2}$$

- **Spectrum** = zero mode + KK excitations of mass $\mathbf{m_n^2}$

Unification

- Suppose typical energy $\mathbf{E} \ll \mathbf{R}^{-1} \rightarrow$ only zero modes can be excited.

- Expand the Einstein action in 5-dim, keeping only zero modes and set $\Phi = 1$

$$\hat{\mathbf{S}}_{\mathbf{G}}(\hat{\mathbf{g}}) = \mathbf{S}_{\mathbf{G}}(\mathbf{g}^{(0)}_{\mu\nu}) + \mathbf{S}_{\mathbf{EM}}(\mathbf{A}^{(0)}_{\mu})$$

- To obtain the correct normalization

$$\mathbf{S}_{\mathbf{G}} \quad \rightarrow \quad \frac{1}{\mathbf{G}_{\mathbf{N}}} \quad = \quad \frac{\int dx_5}{\hat{\mathbf{G}}_{\mathbf{N}}} = \frac{\mathbf{2}\pi\mathbf{R}}{\hat{\mathbf{G}}_{\mathbf{N}}}$$

$$\mathbf{S}_{\mathbf{EM}} \quad \rightarrow \quad \mathbf{k} \quad = \quad \sqrt{\mathbf{16}\pi\mathbf{G}_{\mathbf{N}}}$$

- Gravity and E.M. unified in higher dimensional space!

- **Geometrical meaning of gauge transf.** $d\hat{s}^2$ invariant under local transformations

$$x^5 \rightarrow x^5 - k\Lambda(x), \qquad A^{(0)}_{\mu} \rightarrow A^{(0)}_{\mu} + \partial_{\mu}\Lambda(x)$$

Charge Quantization

- Consider scalar field φ coupled to 5-dim gravity

$$S = \int d^5\hat{x} \sqrt{-\hat{g}}\, \hat{g}_{MN} \, (\partial_M \varphi)\, (\partial_N \varphi)$$

- Each KK mode $\mathbf{n}$ has

mass $= \frac{\mathbf{n}}{\mathbf{R}}$ **charge** $= \frac{\mathbf{nk}}{\mathbf{R}}$

- Determination of fine-structure constant

$$\alpha = \frac{k^2}{4\pi R^2} = \frac{4\mathbf{G_N}}{R^2}$$

$$\mathbf{R} = \sqrt{\frac{4\mathbf{G_N}}{\alpha}} \simeq \mathbf{4 \times 10^{-31} m} \simeq \left(\mathbf{5 \times 10^{17} GeV}\right)^{-1}$$

NOT a Theory of real word!

- Φ dynamical fields $\rightarrow \mathbf{F}^{(0)}_{\mu\nu}\mathbf{F}^{(0)\mu\nu} = \mathbf{0}$

In addition to E.M. forces we observe in nature strong interactions and weak interactions.

- **Weak interactions**

  E.g. **beta decay**

$$\mathbf{n} \to \mathbf{p}\ \mathbf{e}^-\ \bar{\nu}$$

  is described by Fermi theory of local four-fermion interaction

$$S = \sqrt{2}\mathbf{G_F} \int d^4x\ \left(\bar{\psi}_P\ \gamma^\mu_L\ \psi_N\right)\ \left(\bar{\psi}_\nu\ \gamma^\mu_L\ \psi_e\right)$$

  where $\mathbf{G_F}$ has the same dimension of $\mathbf{G_N}$ (in relativistic unities).

$$\sqrt{2}\mathbf{G_F} = \frac{1}{\Lambda^2_{EW}} \qquad \mathbf{\Lambda_{EW}} \simeq\ \mathbf{240\ GeV}$$

- We know now that for energies above Λ_{EW}, Fermi theory is described by dynamical **massive** vectorial gauge fields:

  charged (W) and neutral (Z) WEAK gauge bosons. For $|q^2| \ll M^2_W$, $\frac{-i}{q^2 - M^2_W} \to \frac{i}{M^2_W}$

$$\mathbf{G_F} = \frac{g^2}{8\mathbf{M^2_W}}$$

Standard Model

- **Unification of E.M. and WEAK interactions** is achieved by **non-abelian gauge theory based on gauge group**

$$\mathbf{SU(2)_L \times U(1)}$$

  $\mathbf{SU(2)_L}$ is spontaneously broken by Higgs mechanism to give masses to W, Z.

- **SM** predictions are in impressive agreement with all precision tests at high energy colliders, e.g. LEP, Tevatron, HERA.

- Unification of gauge couplings at

$$E \simeq 10^{16}\ \text{GeV}$$

  (achieved in minimal supersymmetric version of SM)

- **Planck mass**: in analogy with $\mathbf{\Lambda_{EW}}$

$$\mathbf{G_N \simeq \frac{1}{M_{pl}^2} \qquad M_{pl} \simeq 10^{19}\ GeV}$$

- **Why $\mathbf{\Lambda_{EW} \ll M_{pl}}$?**

The Hierarchy Problem

- Two fundamental scales in nature
  - Planck Mass $M_{\text{pl}} \simeq 10^{19}$ GeV
  - Electroweak scale $\Lambda_{\text{EW}} \simeq 240$ GeV
- 17 order of magnitudes of difference !
- Difficult to mantain at quantum level
- Quadratic divergencies in scalar sector

  ($\delta m_H^2 = \alpha \Lambda_{UV}^2 + \ldots$ fine tuning problem)

- Supersymmetry solves this problem

  boson $\leftrightarrow$ fermion symmetry

- Standard approach: Fundamental theory is at M_{pl} and Λ_{EW} is derived as a small number from dynamics

Q-Gravity in Large Extradimensions

A new approach to the hierarchy problem
(Arkani-Hamed, Dimopoulos, Dvali ’98)

Λ_{EW} is a fundamental scale
M_{pl} is a derived effect from geometry

This requires

- **Compact Extra Spatial Dimensions**
- **Confinement of matter on subspaces**

The idea: Matter fields confined in the usual 4 dimensional space, **Gravity propagating in $D = 4 + \delta$ dimensional space**

Weakness of gravity: Due to large compact spatial dimensions

$$G_N = \frac{G_N^D}{V_{D-4}}$$

Newton law modified at small distances

- $r > R$ $$\mathbf{F_N(r) = G_N \frac{m_1 m_2}{r^2}}$$

- $r < R$ $$\mathbf{F_N(r) = G_N R^{\delta} \left(\frac{m_1 m_2}{r^{2+\delta}}\right)}$$

The potential

$$\mathbf{V(r) = G_N \frac{m_1 m_2}{r} \left(1 + C e^{-\frac{r}{\lambda}}\right)}$$

Plank masses

$$G_N = \frac{1}{M_{pl}^2} \qquad G_N^D = \frac{1}{M_D^{D-2}}$$

$$M_{pl} = \frac{1}{\sqrt{8\pi}} M_D \left(R M_D\right)^{\frac{D-4}{2}}$$

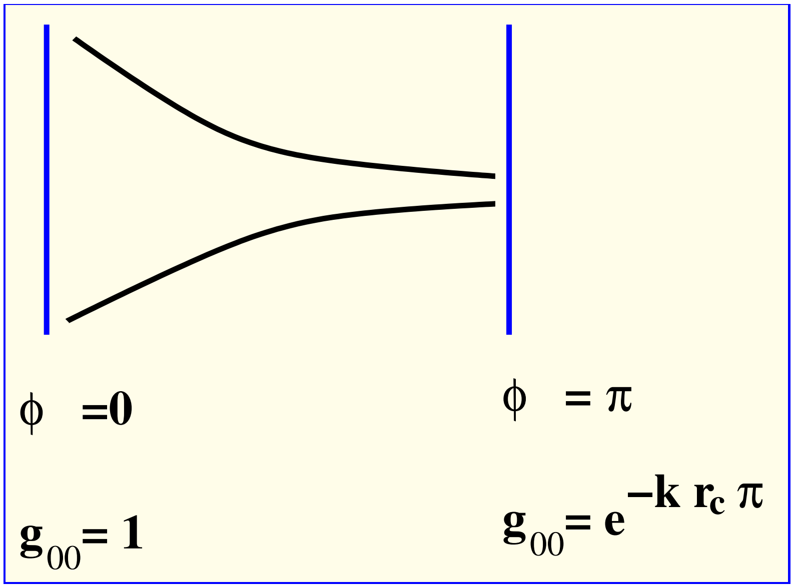
If we want $M_D \simeq 1$ TeV, then R must be large in units of M_D^{-1}.

$D = 4+\delta$, $M_D \simeq 1$ TeV $\Rightarrow$ $R \simeq 10^{\frac{30}{\delta}-17}$ cm

- $\delta = 1$ $\quad R = \left(5 \times 10^{-4}\ \text{eV}\right)^{-1} \simeq 10^8$ **km**
- $\delta = 2$ $\quad R = \left(5 \times 10^{-4}\ \text{eV}\right)^{-1} \simeq 0.4$ **mm**

- **Since Newton law has NOT been tested down the mm scale $\Rightarrow$ $\delta \geq 2$**
- **Strong Gravity at TeV scale for $\delta \geq 2$**

Randall-Sundrum Model



- The Einstein Eqs. in 5 dimension, admits a solution for a **non-factorizable metric**

$$\mathbf{ds^2 = e^{-2kr_c|\varphi|}\ \eta_{\mu\nu}\ dx^{\mu}dx^{\nu} + r_c^2\ d\varphi^2}$$

where $-\pi < \varphi < \pi$

- **Masses related by**

$$\mathbf{\frac{m_\pi}{m_0} = e^{-k\ r_c\ \pi}}$$

- If compactification radius of 5th dim. is $\mathbf{r_c \simeq 10\ k^{-1}}$

$$\frac{\mathbf{m}_\pi}{\mathbf{m_0}} \simeq \frac{\mathbf{M_Z}}{\mathbf{M_{GUT}}}$$

where $\mathbf{M_{GUT}}$ is the scale of Grand Unification $\mathbf{M_{GUT} \simeq 10^{16}}$ GeV

- Masses subject to a Weyl rescaling of a factor $\mathbf{e^{2kr_c\pi}}$.

- If Planck mass in 5-dimensions is of order $\mathbf{M_5 \simeq k \simeq}$ **TeV**

$$\mathbf{M_{pl}^2 = \frac{M_5^3}{k}\left[e^{2kr_c\pi} - 1\right] \gg M_5^2}$$

- Hierarchy problem naturally explained for $\mathbf{r_c \cdot k \simeq \mathcal{O}(10)}$

- Graviton KK modes have large mass gap and are **strongly** coupled.

Kaluza-Klein excitations of Gravity

- Massive KK Gravitons

$$\mathrm{L_{int}(x) \sim \frac{1}{M_{pl}} \Sigma_n G^{(n)}_{\mu\nu}(x) T^{\mu\nu}(x)}$$

- Massive KK Graviscalars

$$\mathrm{L_{int}(x) \sim \frac{1}{M_{pl}} \Sigma_n S^{(n)}(x) T^{\mu}_{\mu}(x)}$$

$\mathrm{T_{\mu\nu}(x)}$ Energy Momentum Tensor

Masses of KK excitations $\mathrm{M_n^2 \sim \frac{n^2}{R^2}}$

- Impossible to detect standard quantum gravity effects for energies of order of $\mathrm{E} < \mathcal{O}(1)$ TeV
- The production of one single KK mode including zero mode is suppressed by terms of order $\mathrm{(E/M_{pl})^2}$.
- Inclusive production of KK modes is huge !

Inclusive production of KK Gravitons

- Mass splitting

$$\Delta \mathrm{m} \simeq \frac{1}{\mathrm{R}} = \mathrm{M_D} \left(\frac{\mathrm{M_D}}{\mathrm{M_{pl}}}\right)^{2/\delta}$$

$\Delta m = 20\text{KeV} \qquad (\delta = 4)$

$\Delta m = 7\text{MeV} \qquad (\delta = 6)$

- For small $\delta \leq 6$:

  Large number of accessible KK modes.

  continuum: $\sum_{\mathrm{n}} \longrightarrow \int \mathrm{dN}$

$$\mathrm{dN} = \mathrm{A}_{\delta-1} \left(\frac{\mathrm{M_{Pl}^2}}{\mathrm{M_D^{2+\delta}}}\right) \mathrm{m_G^{\delta-1} dm_G}$$

  $\mathrm{A}_{\delta-1}$ area of δ-sphere, $\mathrm{m_G}$ graviton mass

- Inclusive production of KK Gravitons

$$(\mathrm{E}/\mathrm{M_{pl}})^2 \quad \Rightarrow \quad (\mathrm{E}/\mathrm{M_D})^{2+\delta}$$

Bounds from Astrophys./Cosmology

SuperNova SN1987A collapse

- Nucleon-Nucleon Brehmstrahlung (T$\simeq$ 30 MeV)

$$\mathcal{L}_{grav} \simeq M_{SN} \times \frac{n_N^3}{\rho} \times (30\text{millibarn}) \times \left(\frac{T}{M_D}\right)^{\delta+2}$$

$$\mathcal{L}_{grav} \leq 10^{53}\text{ergs}^{-1} \simeq (10^{16}\text{GeV})^2 \quad \Rightarrow \quad M_D > 10^{\frac{15-4.5\delta}{\delta+2}}\ \text{TeV}$$

- $\mathbf{M_D > 30}$ **TeV** $(\delta = 2)$ $\mathbf{M_D > 2}$ **TeV** $(\delta = 3)$

Cosmic γ-Rays

$\mathbf{G \to \gamma\gamma}$ $\qquad \tau \simeq 3 \times 10^9\text{yrs}\left(\frac{100\text{MeV}}{m_G}\right)^3$

Gravitons sources

- Thermal production after nucleos :

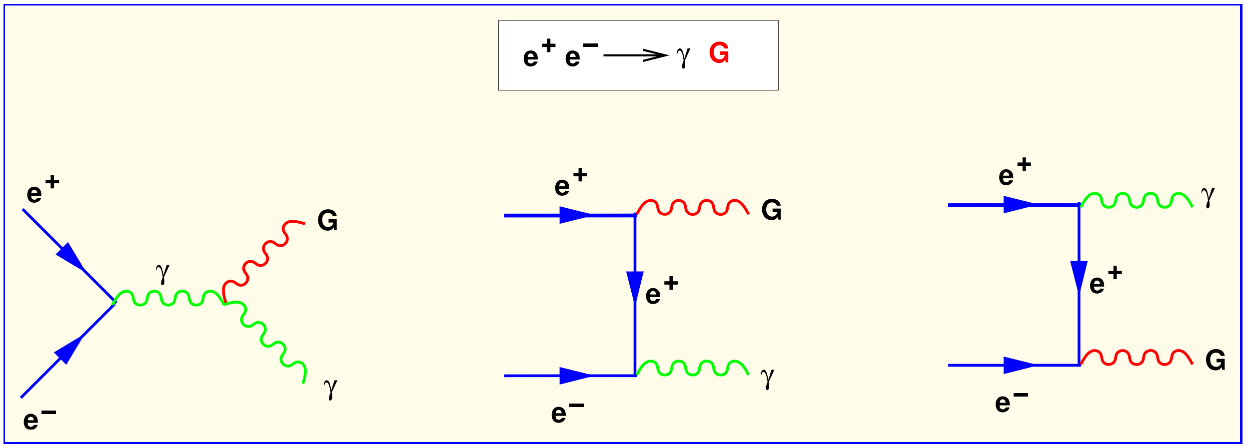
  $M_D > 110$ **TeV** $(\delta = 2)$ $M_D > 5$ **TeV** $(\delta = 3)$

- Neutron stars :

  $M_D > 450$ **TeV** $(\delta = 2)$ $M_D > 30$ **TeV** $(\delta = 3)$

Bounds from accelerator experiments

- KK Gravitons production mechanism at e^+e^- colliders



- Massive gravitons G decay outside the detector

- G detected as missing energy

- Hadron colliders (elementary processes)

$\mathrm{q\bar{q} \to g + G}$ $\quad$ $\mathrm{q\ g \to g + G}$ $\quad$ $\mathrm{g\ g \to g + G}$

- **Precision Observables:**
  - $(g-2)$ of muon M.L. Graesser (2000)
  - precision EW observables at LEP
    ADD ('98)
  - four-fermion interactions
    ADD ('98), G. Giudice, R. Rattazzi, J.D. Wells ('98)

- **Rare decays to Kaluza-Klein Gravitons**
  - $\mu \to \nu_\mu \mathrm{e} \bar{\nu}_\mathrm{e} + \mathbf{G}$ $\quad$ $\mathbf{K} \to \pi + \mathbf{G}$
    $\mathbf{J/Psi} \to \gamma + \mathbf{G}$ $\quad$ $\mathbf{Y} \to \gamma + \mathbf{G}$
    J.Bijnens and M.Maul (2000)
  - $\mathbf{Z} \to \mathbf{f\bar{f}} + \mathbf{G}$ $\quad$ $\mathbf{t} \to \mathbf{bW} + \mathbf{G}$ $\quad$ $\mathbf{H} \to \mathbf{f\bar{f}} + \mathbf{G}$
    $\mathbf{H} \to \mathbf{W^+W^-} + \mathbf{G}$
    E.G. and B.Mele (2002)

Experimental Bounds on M_D

G. Giudice, R. Rattazzi, and J.D. Wells ('98)

discovery requirement $(\sigma_{\text{Signal}} > 5\sqrt{\sigma_B \mathcal{L}}/\mathcal{L})$

$e^+e^- \to \gamma + G$ **G as missing energy**

- **LEP200,** $\mathcal{L} = 4 \times 500\ \text{fb}^{-1}$ **($\sqrt{s}$=200 GeV)**

$\delta = 2 \qquad M_D > 1.3$ **TeV**

- e^+e^- **LC** $\mathcal{L}$**=200 fb**$^{-1}$ **($\sqrt{s}$=1 TeV)**

$\delta = 2 \qquad M_D > 4.1$ **TeV**

$pp \to \text{Jets} + G$ **G as missing energy**

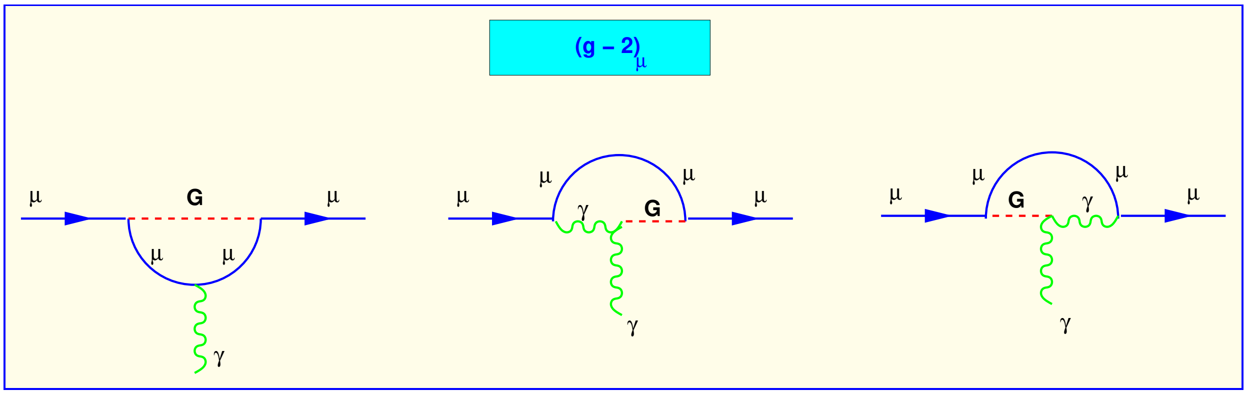
- **LHC ,** $\mathcal{L} = 100\ \text{fb}^{-1}$

$\delta = 2 \qquad M_D > 8.5$ **TeV**

Bounds from precision observables

g-2 of the muon

Graesser (2000)



- It is very sensitive to new physics

- Contribution of each KK graviton to g-2 is finite. $\sum$ over KK modes is divergent!

$$\Delta a_\mu = \frac{7}{12\pi}\left(\frac{\mathbf{m}_\mu}{\mathbf{M_D}}\right)^2 \times \lambda \qquad (\delta = 2)$$

$$\boldsymbol{\Delta a_\mu^{\text{Ext.Dim}} = 2 \times 10^{-9}} \left(\frac{\text{TeV}}{\mathbf{M_D}}\right)^{\mathbf{2}} \times \ \lambda$$

BNL E821 $\Rightarrow \Delta a_\mu < 4 \times 10^{-10}$ at 2σ level

$$\mathbf{M_D > 2.2}\ \text{TeV}\ \sqrt{\lambda}$$

Rare decays to KK gravitons

- Decay $\mathbf{K \to \pi + G}$ ADD ('98)

  Due to angular momentum conservation $\Rightarrow$ spin-0 KK gravitons

$$\Gamma_{K\to\pi+G} \simeq \frac{1}{16\pi}\left(\frac{m_K^5}{m_W^4}\right) \times \left(\frac{m_K}{M_D}\right)^{\delta+2}$$

  $\mathrm{BR} < \mathbf{10^{-12}}$ (for $\delta = 2$ and $M_D = 1$ TeV)

- **Quarkonium($\mathbf{q\bar{q}}$) $\to \gamma$ + G**

  Bijnens and Maul (2000)

  Y decays are more favorable than $J/\Psi \to \gamma + G$

  $\mathbf{Y \to \gamma + G}$ spin-2 dominated

  $\mathrm{BR}^{\mathrm{exp}}(Y \to \gamma + G) < 10^{-5}$

  $\mathbf{M_D > 50 \ \ (9)}$ **GeV** for $\delta = 2$ (6)

Bounds on M_D from Z, W, t, H decays

E.G. and B.Mele, Nucl.Phys. B647, 319 (2002)

$$\mathrm{BR}(\mathrm{Z} \to \sum_{\mathrm{f}} \mathrm{f}\bar{\mathrm{f}} + \mathrm{G_X}) = 8.2 \times 10^{-8} \left(\frac{\mathrm{TeV}}{\mathrm{M_D}}\right)^4 \qquad (\delta = 2)$$

$$\mathrm{BR}(\mathrm{W} \to \sum_{\mathrm{f}',\mathrm{f}} \mathrm{f}'\bar{\mathrm{f}} + \mathrm{G_X}) = 5 \times 10^{-8} \left(\frac{\mathrm{TeV}}{\mathrm{M_D}}\right)^4 \qquad (\delta = 2)$$

$$\mathrm{BR}(\mathrm{t} \to \mathrm{W\,b} + \mathrm{G_X}) = 1.8 \times 10^{-7} \left(\frac{\mathrm{TeV}}{\mathrm{M_D}}\right)^4 \qquad (\delta = 2)$$

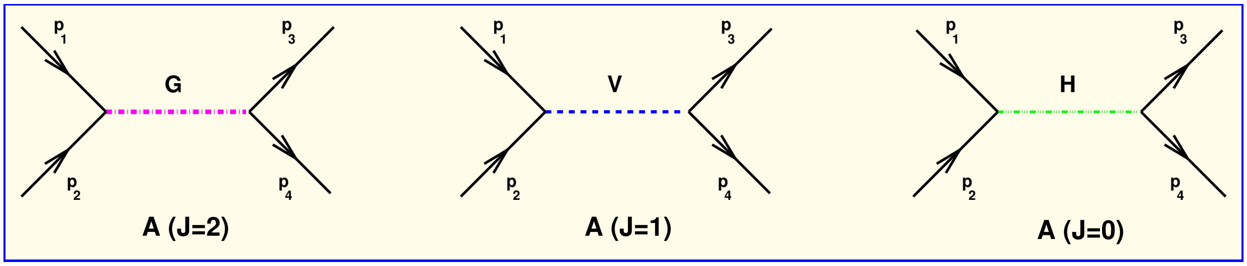
Higgs decays $(\delta = 2)$ $\qquad m_H$

$$\mathrm{BR}(\mathrm{H} \to \bar{\mathrm{b}}\mathrm{b} + \mathrm{G_X}) = 2.2 \times 10^{-7} \left(\frac{\mathrm{TeV}}{\mathrm{M_D}}\right)^4 \qquad 120\ \mathrm{GeV}$$

$$\mathrm{BR}(\mathrm{H} \to \mathrm{WW} + \mathrm{G_X}) = 2.1 \times 10^{-5} \left(\frac{\mathrm{TeV}}{\mathrm{M_D}}\right)^4 \qquad 500\ \mathrm{GeV}$$

Virtual KK Gravitons exchange

Virtual KK Gravitons exchange affect the SM processes



$\mathbf{e^-e^+ \to f\bar{f}}$, $\mathbf{e^-e^+} \to \gamma\gamma$, $\mathbf{W^+W^- \to W^+W^-}$, etc.

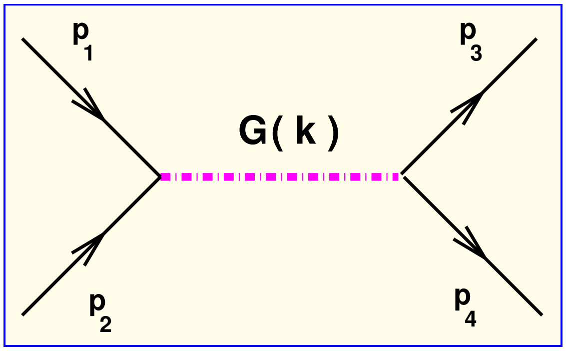
- Real part of amplitude $\mathbf{Re[A]}$ is divergent.
  Predictivity in terms of M_D is lost.

- Imaginary part $\mathbf{Im[A]}$ is finite (due to branch-cut singularity of real gravitons emission)

- Resonant SM processes can have interference with $\mathbf{Im[A]}$ of the graviton exchange
  Finite result predicted in terms of M_D.

- Virtual Graviton exchanges at the Z-pole in large Extra Dimensions
  A. Datta, E.G, and B. Mele, PLB 552 (2003) 237

Scattering amplitude (s-channel)



$$(k^2 = s) = \sum_n \frac{-i}{s - m_n^2}$$

$T_{\mu\nu}$ does not depend on n $\rightarrow$ Factorization

$$\mathcal{A} = \mathcal{S}(s) \left(T_{\mu\nu} T^{\mu\nu} - \frac{1}{\delta + 2} \left(T^{\mu}_{\mu} \right)^2 \right)$$

$\mathcal{S}(s) = \frac{1}{M_{pl}^2} \sum_n \frac{1}{s - m_n^2}$ continuum $\sum_n \rightarrow \int$

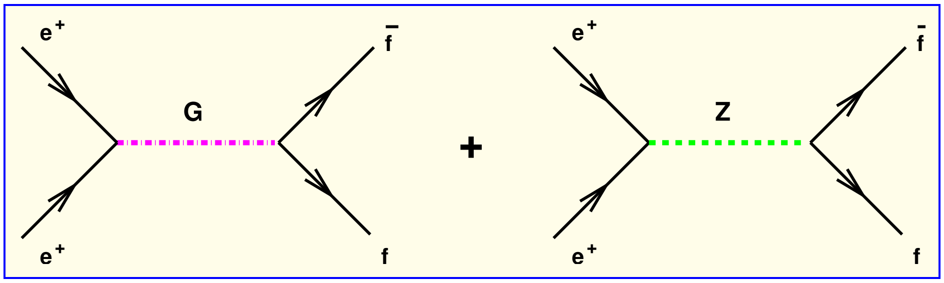
- Real part of $\mathcal{S}$ is divergent

- **Imaginary part of $\mathcal{S}$ is finite**

$$\mathrm{Im}[\mathcal{S}(s)] = \frac{A_{\delta-1}}{2} \frac{s^{\frac{\delta-2}{2}}}{M_D^{2+\delta}}$$

Gravitational interference at Z pole

A. Datta, E.G, and B. Mele, PLB 552 (2003) 237



$$\frac{d\sigma_f}{d\cos\theta} = \frac{9\pi}{2M_Z^2}\left(\frac{\Gamma_e\Gamma_f}{\Gamma_Z^2}\right) S_f(\cos\theta)$$

$$S_f(x) = 1 + x^2 - \Delta_1\left(1 - 3x^2\right) + 2A_eA_f\left(x + \Delta_2 x^3\right)$$

$$\Delta_{1,2} \propto \left(\frac{M_Z}{M_D}\right)^{2+\delta}$$

- **New contribution: vanishes** when integrated over the full range of $\cos\theta = (-1, 1)$
- $\delta = 2$, $M_D = 1$ **TeV** $\Rightarrow \Delta_{1,2} \simeq \mathcal{O}(10^{-4})$ **too small for present EXP. sensitivity**

Virtual KK Gravitons in Higgs production

A. Datta, E.G, and B.Mele, JHEP 0310:003,2003

Light Higgs

- **Hadron Colliders**: $gg \to H \to \gamma\gamma$ **(1-loop)^2**

- Interference with $Im[A(\ gg \to G, S \to \gamma\gamma\ )]$

- Quite small effect, being coupled to the Trace[T^μ_μ]

  For massless final states $T^\mu_\mu \propto \mathcal{O}(\alpha_S, \alpha_e)$ gravitational anomalies.

Heavy Higgs

- $\mathrm{e}^+\mathrm{e}^-$ **Colliders**: $WW \to H \to WW$ **Tree!**

- Large interference $Im[A(WW \to G, S \to WW)]$

  since $T^\mu_\mu \propto M_W^2$

- $\mu^+\mu^-$ **Colliders**: $\mu^+\mu^- \to H \to (WW),\ (\bar{t}t)$

Conclusions

- **Scenarios of large Extra Dimensions offer alternative ways to solve the Hierarchy problem**

- If QG scale (M_D) is in the TeV region, dramatic scenarios with non standard physics are expected at LHC:
  strong gravity at TeV scale, black holes formations.

- Negative search at LEP200 $\mathbf{e^+e^- \to \gamma + G}$ set bounds of order $\mathbf{M_D} \geq 1$ TeV for 2 extra dimensions, comparable with $\mathbf{Z \to f\bar{f} + G}$ from LEP1.

- In perturbative region, **LHC can probe QG scale up to 8 TeV for** $\delta = 2$